ABSTRACT

Today we observe a divide between urban and rural areas with respect to adoption of advanced mobile technologies and services. This can be attributed to various factors. We propose a framework to develop an index to measure this divide. Our two-stage method builds a consensus based on OWA operator in its first stage and uses this consensus to derive a divide measuring index using Graph Theory Matrix Approach in the second stage. The index can be used by different stakeholders such as consumers, policy makers and operators to measure the performance of the operators, which will help in their decision-making.

KEYWORDS: Consensus building, Graph Theory, Mobile Services, Index Development, Ordered Weight Aggregation, Fuzzy Preference

INTRODUCTION

In many developing countries like India, Malaysia and Bangladesh, 3G and 4G/LTE are still at their initial stage of adoption. Since, Mobile Network Operators (MNOs) are interested in quick return on investment, they primarily start targeting urban users first. This creates an information access divide among the urban and the rural users. There are several factors that play a critical role in the adoption or abandonment of advanced mobile technologies and services.

This paper first identifies few relevant factors and then derives an index, which can be used to measure this divide. Thus, this paper has primarily three contributions: first, a precise list of factors are identified; second, a consensus among the experts are formed using our proposed consensus building algorithm and finally, a systematic approach to derive an index is recommended. The index can be used to compare the performance of different operators, and thus, it can help in improving the overall experience of using the services at the desired location, help in promoting healthy competition among the operators and can help in improving the existing policies.

The flow of the paper is as follows: Section 2 presents the background for the need for such framework and related concepts are discussed; the proposed framework is presented in Section
3. An illustrative example is provided in Section 4, finally, conclusions drawn are presented in Section 5.

BACKGROUND

While solving real life problems, decision-making is not so straightforward and most of the decisions are made under uncertainty. In most of the cases, there is no direct way to take a firm decision. Hence, opinions are taken from the experts. It is a well-acknowledged fact that the ability to take a virtuous and timely decision under dubious situation helps an organization to gain competitive advantage. However, there are certain problems associated with the collection of opinion from the experts such as bringing all experts under one table, the dominance of a senior or influential person leading to a biased decision making.

In the era of Internet and social media, the traditional barrier to bringing experts’ on a round table has almost become extinct as virtual meetings have become more common. There are situations when the experts have different opinions on the same subject, and decision-making becomes a difficult task, especially when there is a lack of common framework. In this situation, often, the entire process becomes disorganized resulting in a biased decision.

Normally, board members, who are usually comprised of more than one person, take decisions. Similarly, multiple factors are considered when the decision is to be taken. It may happen that the opinion on a particular note may differ. In the absence of appropriate consensus-building algorithm, decisions may be taken without much deliberation leading to bias in the decision-making. As a result, the decision may not incorporate suggestions of the expert having a different opinion. In such situations, a guided framework can assist in incorporating opinions of each expert in the decision-making process. Lack of such a framework may lead to bias in decision-making and conflict may arise among the experts involved in the process. In order to address this issue, several researchers (Herrera-Viedma et al., 2002; Chen and Lin, 2004; Choudhury et al., 2006; Singh et al., 2007) have proposed different consensus building algorithms that address this problem.

Consensus usually involves compromise, but at the same time, it is considered safe, as none of the experts are 100% sure about the future events. Rather they share their expertise so that decisions on critical factors can be taken. It may help in being prepared for possible events that may occur in the future. It is also well acknowledged and understood that a group will lead to a better decision under uncertainty than just an individual (Herrera-Viedma et al., 2002; Choudhury et al., 2006; Singh et al., 2007). The phenomenon where more than one person participates in the decision-making process is termed as Group Decision Making (GDM). GDM is a process of converging to an agreement on a common theme when more than one person participate. It helps to minimize the decision bias, often created by an individual, in a decision-making process. A group decision is usually carried out in an unstructured manner and requires substantial time to reach a consensus. The GDM process gets complicated as multiple criteria are taken into account as it involves several subjective and qualitative factors. Further, the knowledge of the members may vary, which may lead to a conflict.

A framework that satisfies the group and is capable of eliminating the dominance of an individual such that a reliable decision can be taken by minimizing the bias is of much help. Conventional approaches such as Delphi and Nominal Group Technique normally fail to deliver a fair decision. In these approaches, often, senior member overtakes the decision-making process and individual inclination becomes quite common phenomena during consensus
building process. It leads to unfair estimation about the intensity of the problem. In order to overcome some of the mentioned problems and limitations of existing consensus building approaches a guided framework that can smoothen the entire decision-making process is proposed.

**Consensus Building**

Consensus is typically defined as the full and unanimous harmony among all the experts regarding all the feasible alternatives (Herrera-Viedma et al., 2002). However, forming a full agreement on any issue is not possible in a complex real-life situation. In simple terms, consensus mainly refers to how any group of experts who value autonomy might work together. A decision that involves consensus might take an exceptionally longer time, and thus may be unbearable for critical matters, e.g., decisions that involve competitive advantage or strategic policy.

Before going into a detailed explanation of the consensus building process, first we define some of the commonly referred terms such as entropy, consensus measure, proximity measure, and confidence degree and consensus level. Now, we define each term one by one.

**Consensus Measure** has been defined as "the measure of agreement amongst the experts involved in decision-making process". It guides experts to form an agreement, possibly over little iteration. Different distance metrics is used to evaluate the difference of opinions (Herrera-Viedma et al., 2002; Choudhary et al., 2006).

**Proximity Measure** is defined as "the measure of agreement amongst the expert's opinion with the collaborative solution". The measure of proximity provides feedback to the expert so that next iteration is closer to the solution (Herrera-Viedma et al., 2002; Choudhary et al., 2006).

**Consensus Level** is the defined as "the agreed upon value to terminate the consensus process". It means that all the experts have reached to the agreed upon the level and the iteration process can be terminated (Herrera-Viedma et al., 2002; Choudhary et al., 2006).

**Entropy** (also termed as dispersion) is one of the measures derived from Shannon entropy that calculates the variability in the opinions furnished by the experts for the considered problem (Shannon, 1948).

The entropy of the system, \( Ent(X) \) is calculated using the formula given below:

\[
Ent(X) = - \sum_{i=1}^{n} p_i \log_2(p_i)
\]

Where, \( X \) is the opinion of different experts. The Shannon entropy takes into account the probabilities \( p_i \) and \( n \), the number of opinions from different experts.

**Formats for Recording Opinion**

An expert can give their opinion in a format in which he/she is more comfortable. It could be possible that the experts may not be comfortable with the strict format that we might prefer to build consensus. There are different formats that can be used to record the opinion of the
experts as suggested by Herrera-Viedma et al., (2002), which is discussed below. Hence, four different formats (multiplicative preference, utility values, fuzzy preference and preference ordering) are made available for the experts so that they can furnish their opinions in their preferred format.

**Multiplicative Preference Structure**: In multiplicative preference structure, the values are assigned with the help of pair-wise comparisons. The upper triangular matrix is the inverse of the lower triangular matrix in the multiplicative preference structure input format. The expert \( \delta_e \) assign the values on a scale of 1 to 9 (Saaty, 2005). For example, if the value associated with factor \( \alpha_i = 7 \), then \( \alpha_i = 1/7 \).

**Utility Values**: The expert \( \delta_e \), provides his/her input based on a set of \( n \) factors in terms of utility value. Utility values are represented as \( u^e = \{ u^e_i, i=1,2,...,n \} \); \( u^e_i \in [0,1] \). \( u^e_i \) represents the utility associated with the factor \( e \) for the factor \( \alpha_i \) (Herrera-Viedma et al., 2002; Choudhary et al., 2006). If the value associated with the factor \( \alpha_i \) is high then, it is considered to be more important than others.

**Fuzzy Preference Relations**: In this case, the expert \( \delta_e \) assigns his/her preferences on a set of alternatives \( \alpha \) using a fuzzy membership function \( \mu_e^{\alpha} \); it is stored in matrix form \( F^e \), where, \( e \) is the expert. \( F^e \) is a set of all alternatives represented as \( F^e \subset \alpha \times \alpha \). The fuzzy membership function is represented as \( \mu_e^{\alpha} : \alpha \times \alpha \rightarrow [0,1] \) where, \( \mu_e^{\alpha}(\alpha_i, \alpha_j) = f_{ij}^e \) the preference degree or intensity of the factor \( \alpha_i \) over \( \alpha_j \). \( f_{ij}^e \) is the fuzzy preference degree given by \( e \)th expert over alternatives \( \alpha_i \) and \( \alpha_j \) (Herrera-Viedma et al., 2002; Choudhary et al., 2006). In this case, it is fair to assume that \( f_{ii}^e + f_{jj}^e = 1 \) and \( f_{ij}^e = 1/2 \) in particular case.

**Preference Ordering for the Factors**: In preference ordering, as the name suggests, the expert \( \delta_e \) decides the order among the set of factors \( \alpha \). The preference ordering \( (\rho^e) \) is a set of values given by \( e \)th expert represented as \( \rho^e = \{ \rho_1^{(e)}, \rho_2^{(e)}, ..., \rho_n^{(e)} \} \) where, \( \rho_i^{(e)} \) is the preference assigned for factor \( \alpha_i \) by an expert \( e \) (Herrera-Viedma et al., 2002; Choudhary et al., 2006).

**Graph Theoretic Approach**

Graph theory is a well-established branch of mathematics and computer science. The concept was first introduced and applied to represent the Seven Bridges of Konigsberg by Euler in 1736. Since then, it has been applied in several disciplines to solve numerous problems ranging from computer science, biology to decision-making. It represents the problem in the form of vertices and edges. Sometimes, the edges are associated with weights. These weights ascertain the importance of connections among the pair of vertices.

Mathematically, a graph is represented as \( G = (V,E,\lambda) \). It consists of a set of objects \( V=\{v_1, v_2,..., v_n \} \) called vertices; another set \( E=\{e_1, e_2, ..., e_n \} \) whose elements are called edges, such that unordered pair of \( (v_i, v_j, e_k) \) vertices identifies edges. \( \lambda \) maps weights associated with edge and vertex connections (Deo, 1974). A graph can be directed or undirected. For a detailed discussion on graph theory concepts, the researcher is suggested to refer the book written by Deo, (1974).

**Graph Theoretic Matrix Approach (GTMA)**

According to Grover et al. (2004), and Sabharwal and Garg (2013), digraph and matrix approach is used to represent the problem in the form of vertices and edges, where matrix
representation of the graph helps in faster processing through computer algorithms. It takes into account structural as well as the functional interaction of the overall system. In addition, GTMA has several advantages over other approaches such as case studies, empirical studies, structural equation modeling and multi-criteria decision-making approaches. It presents a more appropriate mechanism to obtain a single score for all the factors under consideration. It follows a systematic method for conversion of qualitative variables to a quantitative value. It is based on the isomorphic properties of the two graphs. It leads to self-analysis (comparing with the best case or desired level) and comparison of a similar context within or outside the organization.

PROPOSED FRAMEWORK

As a first step, critical success factors are identified. These factors are then represented as a graph, which is stored in a matrix containing the off-diagonal elements, whereas the diagonal elements represent weights associated with the vertices. The edge weights are the visual representations of the characteristics and their inter-dependencies on other factors, whereas the weights associated with the vertex represents the importance of the factor in the overall system. These weights are obtained with the help of participating experts or from the available data, which is frozen with the help of consensus-building algorithm discusses below. These edge weights, which is stored in the matrix is used to obtain the index of the system under study. The matrix converts the graph into a mathematical form to obtain the index by calculating the permanent matrix. The obtained score is termed as Mobile Service Divide Index (MSDI). Contrary to the determinant, permanent has no negative symbol, which means there is no loss of information. Hence, application of permanent concept leads to a better estimation of the impact of the factors (Jurkat and Ryser, 1966). The abstract representation of MSDI is as follows:

$$MSDI = f(critical\ success\ factors)$$

MSDI is obtained by calculating the permanent value of the matrix, which consists of the diagonal and off-diagonal elements. The diagonal and off-diagonal values are obtained separately in two different separate steps. In the two-stage process, the first stage deals with forming of consensus among the diagonal elements and off-diagonal elements, whereas the second stage deals with the development of the permanent matrix by combining diagonal and off-diagonal elements. Once, MSDI is calculated, resemblance ratio can be calculated to compare two different scenarios. The proposed algorithm to obtain MSDI is pictorially shown in Figure 1.

We use Yager's neat Ordered Weighted Averaging (OWA) operators for aggregating the opinions of the experts and then apply the concepts of GTMA to obtain MSDI. The opinions for the diagonal elements (i.e., factors) are captured using the linguistic-based qualitative measure (Table 1).

The difference of opinions among the experts is measured using Entropy (Choudhury et al., 2006; Singh et al., 2007). However, Shannon's entropy measure has few limitations. First, it fails to distinguish between two different sets of opinions. Second, the entropy remains constant notwithstanding the order of the grouping of the distribution. Third, neither the mean and standard deviation nor measure of entropy is adequate to capture proximity in ordinal or fuzzy
scales. Therefore, a consensus is reached for the diagonal elements using a modified Shannon's approach as suggested in Tastle and Wierman (2007).

First, it overcomes the problem associated with the entropy measure, and second, the consensus measure remains unaffected with the increase in the size of the number of participants.

The consensus for the diagonal elements is measured using the following equation:

\[
Cons(X) = 1 + \sum_{i=1}^{n} p_i \log_2 \left( 1 - \frac{|X_i - \mu_X|}{d_X} \right)
\]

where, \(X\) is the opinion of different decision makers, \(\mu_X\) is the mean of \(X\) and \(d_X\) is the width of \(X\); \(d_X = X_{\text{max}} - X_{\text{min}}\).

**Steps for Obtaining Index**

In this section, we discuss the steps required to derive the index. In Stage I, helps to build consensus whereas Stage II, calculates the permanent value of the matrix, which is termed as MSDI.

<table>
<thead>
<tr>
<th>Linguistic-based qualitative measure</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptionally Low</td>
<td>0.045</td>
</tr>
<tr>
<td>Extremely Low</td>
<td>0.135</td>
</tr>
<tr>
<td>Very Low</td>
<td>0.255</td>
</tr>
<tr>
<td>Low</td>
<td>0.335</td>
</tr>
<tr>
<td>Below Average</td>
<td>0.410</td>
</tr>
<tr>
<td>Average</td>
<td>0.500</td>
</tr>
<tr>
<td>Above Average</td>
<td>0.590</td>
</tr>
<tr>
<td>High</td>
<td>0.665</td>
</tr>
<tr>
<td>Very High</td>
<td>0.745</td>
</tr>
<tr>
<td>Extremely High</td>
<td>0.865</td>
</tr>
<tr>
<td>Exceptionally High</td>
<td>0.955</td>
</tr>
</tbody>
</table>
STAGE I: Consensus Building

In this stage, several steps to reach consensus is discussed.

Step 1: Opinion Collection
Let α = {α₁, α₂, ..., αₙ} be a set of n factors, and δ = {δ₁, δ₂, ..., δₘ} be m experts, where both m and n are finite. The experts are free to record their opinions in any of the specified four forms. For obtaining the importance of diagonal elements, opinions are recorded based on the linguistic quantifiers (Table 1), which are further mapped to the equivalent fuzzy number. The 11-point fuzzy scale has been used in few studies for recording the opinions of the experts (Venkatasamy and Agrawal, 1997; Rao, 2006). Figure 2 shows the 11-point fuzzy scale conversion mechanism and mapping between X and μₓ to determine equivalent fuzzy values.

![Figure 2: 11-point Linguistic Terms Scale for Conversion to Fuzzy Numbers](image)

Step 2: Input Transformation

Initial responses are collected in various formats, which are then converted to fuzzy preference relations. It is achieved using the following transformation function: Let α = {α₁, α₂, ..., αₙ} be the set of factors that are to be considered for decision making where, αᵢ is the value associated with factor i. Let χᵢ be the evaluation associated with factor αᵢ for the expert δₑ. Then, the intensity of preference of alternative αᵢ over αⱼ, fₑᵢⱼ for δₑ is given by the following transformation function

\[ f_{ᵢⱼ}^ₑ = \phi(χᵢ, χⱼ) \]

\[ f_{ᵢⱼ}^ₑ = \frac{1}{2} [1 + g(χᵢ, χⱼ) - g(χᵢ, χⱼ)] \]  \hspace{1cm} (4)

where g ascertains that:

\[ g(χ, χ) = \frac{1}{2} \forall χ ∈ ℝ \]

and, g is a non-decreasing in the first parameter and non-increasing in second parameter.
A) to bring preference ordering to fuzzy preference relations structure:

Suppose \( \lambda_i^e = \tau_i^e \) represents the preference ordering of factor \( \alpha_i \) and \( g(l, u) \) is a transformation function (Chiclana et al., 1998; Tanino, 1988); \( u \) and \( l \) are the upper and lower bounds and \( n \) is the number of alternatives. Then,

\[
f_{ij}^e = h^1(\tau_i^e, \tau_j^e) = \frac{1}{2} \left[ 1 + \frac{(\tau_j^e - \tau_i^e)}{n - 1} \right]
\]

B) to bring utility values to fuzzy preference relations format:

In this case, if \( \lambda_i^e = v_i^e \) represents the utility value of the factor \( \alpha_i \) and \( g(l, u) \) is a transformation function (Chiclana et al., 1998; Tanino, 1988); \( u \) and \( l \) are the upper and lower bounds and \( n \) is the number of alternatives. Then,

\[
f_{ij}^e = h^2(v_i^e, v_j^e) = \frac{(v_j^e)^2}{(v_j^e)^2 + (v_i^e)^2}
\]

C) to bring multiplicative preference relation to fuzzy preference relations format:

\[ \eta^e = \left( \eta_{ij}^e \right) \] is the association in multiplicative preference relation form. Then, the corresponding additive fuzzy preference relation \( F^e = (f_{ij}^e) \) associated with \( \eta^e \) is given as follows (Saaty, 1980):

\[
f_{ij}^e = h^3(\eta_{ij}^e) = \frac{1}{2} (1 + \log_3 \eta_{ij}^e)
\]

Step 3: Aggregation Operation

In this phase, a collective solution is obtained by using the opinions furnished by the experts after transforming them to the fuzzy preference relations format. The aggregation operation is carried out with the help of neat Ordered Weight Aggregation (OWA) operators (Yager, 1988). Fuzzy linguistic quantifiers like “majority”, “at least half”, “as many as possible”, etc. are used. These quantifiers are defined by providing a lower and an upper bound value, for example, (l, u) can be (0, 0.5) for “at least half”, (0.5, 1) for “as many as possible” and (0.3, 0.8) for representing “majority”. The collective solution is found using the following equation:

\[
f_{ij}^e = \Upsilon_{\mu}(f_{ij}^1, f_{ij}^2, \ldots, f_{ij}^m) = \sum_{e=1}^{n} w_e \cdot f_{ij}^e
\]

where, \( \mu \) is a fuzzy linguistic quantifier that represents the concept of fuzzy majority (Zadeh, ...
1983). It is used to calculate the weights of \(Y\), \(w = \{w_1, w_2, \ldots, w_n\}\) such that, \(w_e \in [0,1]\) and \(\sum_{e=1}^{n} \). 

\[
w_e = \mu\left(\frac{e}{n}\right) - \mu\left(\frac{e-1}{n}\right)
\]

where, \(e = 1, 2, \ldots, n\) and \(\mu(t) = \begin{cases} 0, & \text{if } t < l \\ \frac{t-l}{u-l}, & \text{if } l \leq t \leq u \text{ and } (l, t, u) \in [0, 1] \\ 1, & \text{if } t > u \end{cases}\)

**Step 4: Estimating the Proximity**

In this step, the closeness or the proximity of decisions among different experts is calculated. Thereafter, choice degrees are calculated. The quantifier guided dominance degree (QGDD) is used to quantify the dominance of \(\alpha_i\) over all other factors, and quantifier guided non-dominance degree (QGNDD) provides the information on the non-dominance among remaining factors to calculate the priority of the factors (Orlovsky, 1978). Each choice degree is self-sufficient to rank the factors. However, when there is a difference, the maximum from both is taken to rank them.

\[
QGDD_i = \Upsilon_\mu(f_{ij}^e, j = 1, 2, \ldots, n) \tag{10}
\]

\[
QGNDD_i = \Upsilon_\mu(1 - \max\{f_{ij}^e - f_{ji}^e, 0\}) \tag{11}
\]

The temporary result is stored in \(X_{trreg}\), where

\[
X_{trreg} = \max\{QGDD_i, QGNDD_i\} \tag{12}
\]

**Step 5: Computation of Difference of Opinion**

At the beginning of this process, the number of iterations (N_{ITR}) and the Consensus Level (CL) are pre-decided. The Euclidean distance measure is used to find the difference in opinion furnished by the experts. It is then used to compare with the aggregated collective solution. \(d(v, v^c)\), is the Euclidean difference function where, \(v = (v_1^i, v_1^c, \ldots, v_n^i)\) is the set of individual opinions for all the factors, and \(v^c = (v_1^c, v_1^c, \ldots, v_n^c)\) is the set of collective opinion.

**Step 6: Calculation of Consensus Degree**

The consensus degree \(\zeta(\alpha_j)\) of all the experts on each of the alternative \(\alpha_i\) is calculated using:

\[
\zeta(\alpha_j) = 1 - \sum_{i=1}^{m} \frac{\pi_{iz}(\alpha_j)}{m} \tag{13}
\]
where, $\pi_i(\alpha_j)$ is the proximity degree $\pi_i$ of each factor $\alpha_j$ and $m$ is the number of experts.

**Estimation of Proximity Degree**

The proximity degree $\pi_i(\alpha_j)$ of each expert is calculated for each alternative:

$$\pi_i(\alpha_j) = p(v^i, v^e)(\alpha_j) = f(|v^e_j - v^i_j|)$$  \hspace{1cm} (14)

where, $f(x) = \left(\frac{|v^e_j - v^i_j|}{n-1}\right) ^ \lambda \in [0, 1]$ and $\lambda \in [0, 1]$

Different values for $\lambda$ such as 1, 0.7 and 0.5, etc. can be chosen where $\lambda$ controls the consensus rigorousness. If $\lambda$ is close to 1, then the rigorousness process decreases.

**Step 7: Calculation of Consensus Measure**

The consensus measure $\zeta_X$ is calculated by applying neat OWA operator (Yager, 1988; Yager, 1994). The neat OWA operator for each factor $\alpha_i$ with associated weight

$$w_j = \frac{\alpha_i^j}{\sum_j \alpha_i^j},$$

is defined as

$$\zeta_X = neat\text{OWA}_{OR-LIKE}(\zeta(\alpha_1), \zeta(\alpha_2), \ldots, \zeta(\alpha_n))$$  \hspace{1cm} (15)

$$F(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{\sum \alpha_i^{\beta+1}}{\sum \alpha_i^{\beta}}$$  \hspace{1cm} (16)

where, $\beta$ controls the behavior of the aggregation process. Thus, by using this kind of function, the weights are directly deduced from the values to be aggregated. If $\beta = 0.0$, we get a simple average operator, and as $\beta$ approaches infinity, we obtain a maximum operator. The detailed analysis for neat OWA operators and the effect of $\beta$ on the optimism degree can be found in Zarghami et al. (2008). The sensitivity of neat OWA operator with the OR-LIKE feature is presented in Table 2

<table>
<thead>
<tr>
<th>Fuzzy Linguistic Quantifiers</th>
<th>Quantifier</th>
<th>Optimism degree, $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>At least one of them</td>
<td>$\beta \rightarrow 0.0$</td>
<td>0.999</td>
</tr>
<tr>
<td>Few of them</td>
<td>0.1</td>
<td>0.909</td>
</tr>
<tr>
<td>Some of them</td>
<td>0.5</td>
<td>0.667</td>
</tr>
<tr>
<td>Half of them</td>
<td>1.0</td>
<td>0.500</td>
</tr>
<tr>
<td>Many of them</td>
<td>2.0</td>
<td>0.333</td>
</tr>
<tr>
<td>Most of them</td>
<td>10.0</td>
<td>0.091</td>
</tr>
<tr>
<td>All of them</td>
<td>$\beta \rightarrow \infty$</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Step 8: Calculation of Proximity Measure

The proximity measure $\Pi_X^i$ of the $i^{th}$ decision maker is calculated using the neat OWA_{AND-LIKE} operator, which is $1 - \text{neat OWA}_{OR-LIKE}$.

$$\Pi_X^i = 1 - \text{neat OWA}_{OR-LIKE}(\pi_i(\alpha_j))$$  \hspace{1cm} (17)

Step 9: Re-evaluation Rules

a) If the difference in the rank of opinions between the collective solution and individual solution of the factors is positive, then the concerned expert is requested to change his opinion by lowering the previous values.

b) If it is negative, the expert is requested to increase the evaluation associated with a particular factor.

c) If there is no difference in opinion between the collective and individual solution, previous values are kept.

Step 10: Check for the number of iterations

Step 11: Calculate the fuzzy preference values of interactions among the factors for the micro variables or sub-factors, if present.

STAGE II: Estimation using Permanent Value

Step 12: Combining the Diagonal Matrix and Off-Diagonal Matrix

The permanent matrix $f^c$ is a combination of two matrices. A diagonal matrix $l$ describing the importance of individual factor $l_i$ in the system under consideration and another off-diagonal matrix $f_{sol}$, which contains pair-wise comparison among various factors.

Step 13: Calculate the permanent matrix

Let $A$ be an $n \times n$ matrix. $S_n$ is the symmetric group of degree $n$. If $\sigma \in S_n$, then the set $
abla \{ a_{1\sigma(1)}; a_{2\sigma(2)}; \ldots; a_{n\sigma(n)} \}$ is called a diagonal of $A$ corresponding to the permutation $\sigma$. The product $\prod_{i=1}^{n} a_{i\sigma(i)}$ is called a diagonal product. Following equation is used to calculate the value of the matrix:

$$\text{perm}(M) = \sum_{\sigma \in S_n} \prod_{i=1}^{n} a_{i\sigma(i)} \hspace{1cm} (18)$$

Step 14: Calculate Resemblance Ratio, if the comparison is to be done with other scenarios
Resemblance Ratio (RR) is a measure that helps to compare two different scenarios based on the index obtained in the previous step using Equation 18. The resemblance ratio lies in between 0 to 1. If both companies have same capabilities (regarding handling factors related to mobile service adoption) then the resemblance will be 0. The closer is to 1, and then higher is the non-resemblance. RR can be calculated using the following equation:

$$RR = \frac{1}{\max \left( \text{perm}(M_1), \text{perm}(M_2) \right)} \left( |\text{perm}(M_1) - \text{perm}(M_2)| \right)$$

where, $M_1$ and $M_2$ are the permanent matrix for scenario 1 and scenario 2, respectively.

**AN ILLUSTRATIVE EXAMPLE**

Spectrum is considered as a key resource for the success of mobile services. It makes communication possible over the mobile phones at a specific frequency range. Indian government auctioned 71 blocks of 3G spectrum (1959 - 1979 MHz) in 22 service areas covering pan-India in May 2010 (DOT, 2010). 3G spectrums allow mobile phones to connect to the Internet wirelessly, and the speed of data transfer is high as compared to data speed offered through 2G spectrums.

In order to remain operational in the market, the MNOs retain existing spectrum (2G) and also obtain licenses for new spectrum (3G, 4G/LTE) to float their services. However, there are certain challenges associated with recovering the return on investment.

However, due to huge competition and government policies, basic services like voice and SMS are not able to contribute to their profitability anymore. In order to overcome this, MNOs try to invest and market in mobile services as it is considered to be one of the profitable business units. Despite their efforts, people at large are not adopting them. Several researchers (Carlsson et al., 2005; Bouwman et al., 2007; Nikou and Mezei, 2013) have discussed the possible impact of factors as well as driving forces related to usage of different mobile value added services. In order to address these issues, we have identified few factors that lead to the poor adoption of advanced mobile services. Estimating the cumulative impact of factors can help MNOs and other stakeholders to compare their performance with their previous performances and/or with their competitors.

For the purpose of illustration, we consider only five critical factors, though others can also be considered for detailed analysis. The factors are enumerated below:

1. Operational Cost
2. Low Spectrum Availability
3. Unavailability of Context-aware Services
4. Return on Investment for the operator
5. Inefficient Bundling Strategies

These five factors $I_i$ (i.e., $I_1$, $I_2$, $I_3$, $I_4$, and $I_5$) were circulated to four experts (DM1, DM2, DM3, and DM4). They were requested to assign importance to each of the factors $I_i$ on the overall system. These importance were recorded using 11-point fuzzy scale (Table 1). The weights of each element $I_i$ were aggregated using neat OWA operator (Table 3).

<table>
<thead>
<tr>
<th>DM1</th>
<th>DM2</th>
<th>DM3</th>
<th>DM4</th>
<th>$\mu_{x_i}$</th>
<th>Entropy($x_i$)</th>
<th>Cons($x_i$)</th>
<th>neat OWA$_{Q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\beta = 0.0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>&quot;At least one of them&quot;</td>
</tr>
<tr>
<td>I1</td>
<td>0.7450</td>
<td>0.6650</td>
<td>0.7450</td>
<td>0.6650</td>
<td>0.7050</td>
<td>1.0000</td>
<td>0.9351</td>
</tr>
<tr>
<td>I2</td>
<td>0.3350</td>
<td>0.4100</td>
<td>0.2560</td>
<td>0.3350</td>
<td>0.3338</td>
<td>1.5000</td>
<td>0.9348</td>
</tr>
<tr>
<td>I3</td>
<td>0.0450</td>
<td>0.0450</td>
<td>0.1390</td>
<td>0.0450</td>
<td>0.0675</td>
<td>0.8113</td>
<td>0.9451</td>
</tr>
<tr>
<td>I4</td>
<td>0.5900</td>
<td>0.6650</td>
<td>0.7450</td>
<td>0.6650</td>
<td>0.6663</td>
<td>1.5000</td>
<td>0.9348</td>
</tr>
<tr>
<td>I5</td>
<td>0.4100</td>
<td>0.3350</td>
<td>0.3350</td>
<td>0.1350</td>
<td>0.3038</td>
<td>1.5000</td>
<td>0.8560</td>
</tr>
</tbody>
</table>

Aggregation operation (neat OWA$_{Q}$) was applied on the collected opinions of all elements $I_i$ for different values of $\beta$. It helped to analyze the sensitivity of the neat OWA operator. It was observed that as the value of $\beta$ increased it acted more like OR operation (termed as OR-LIKE) and if $\beta$ was 0.0, it acted more like mean $\mu_x$ operation. Similarly, if $\beta$ was assigned to 1.0, it meant that all participating experts agree to these values up to 50%. Further, we adopted quantifier Q “Half of them” with $\beta = 1.0$ for further processing as it acts as a mean operator. Hence, the aggregated opinion for diagonal matrix I came to be {($I_1$, 0.7617), ($I_2$, 0.3427), ($I_3$, 0.5487), ($I_4$, 0.7307), ($I_5$, 0.7072)}.

In the second part, opinions were collected for the non-diagonal elements in various formats for pairwise comparison and it was then transformed using an appropriate transformation function into fuzzy preference relations’ format. The values obtained from different experts (DM1 and DM3 in fuzzy preference relations format and DM2 and DM4 in multiplicative preference format) are as follows:

$$DM1 = \begin{pmatrix}
- & 0.75 & 0.70 & 0.55 & 0.55 \\
0.25 & - & 0.40 & 0.30 & 0.25 \\
0.30 & 0.60 & - & 0.40 & 0.30 \\
0.45 & 0.70 & 0.60 & - & 0.50 \\
0.45 & 0.75 & 0.70 & 0.50 & -
\end{pmatrix}$$
The initial responses of DM1, DM2 and DM4 are then converted to the fuzzy preference relations P2 and P4 using Equation 7.

$$P1 = \begin{pmatrix} - & 0.75 & 0.70 & 0.55 & 0.55 \\ 0.25 & - & 0.40 & 0.30 & 0.25 \\ 0.30 & 0.60 & - & 0.40 & 0.30 \\ 0.45 & 0.70 & 0.60 & - & 0.50 \\ 0.45 & 0.75 & 0.70 & 0.50 & - \end{pmatrix}$$

$$P2 = \begin{pmatrix} - & 1.0000 & 0.8662 & 0.7500 & 0.6577 \\ 0.0000 & - & 0.3423 & 0.2500 & 0.1845 \\ 0.1338 & 0.6577 & - & 0.2500 & 0.3423 \\ 0.2500 & 0.7500 & 0.7500 & - & 0.2500 \\ 0.3423 & 0.8155 & 0.6577 & 0.7500 & - \end{pmatrix}$$

$$P3 = \begin{pmatrix} - & 0.75 & 0.8 & 0.7 & 0.6 \\ 0.25 & - & 0.3 & 0.3 & 0.2 \\ 0.2 & 0.7 & - & 0.4 & 0.3 \\ 0.3 & 0.7 & 0.6 & - & 0.4 \\ 0.4 & 0.8 & 0.7 & 0.6 & - \end{pmatrix}$$

$$P4 = \begin{pmatrix} - & 0.7500 & 0.8662 & 0.8155 & 0.7500 \\ 0.2500 & - & 0.3423 & 0.2500 & 0.1338 \\ 0.1338 & 0.6577 & - & 0.3423 & 0.8662 \\ 0.1845 & 0.7500 & 0.6577 & - & 0.3423 \\ 0.2500 & 0.8662 & 0.1338 & 0.6577 & - \end{pmatrix}$$

Now, aggregation operation was performed as described in Step 3. Using the fuzzy majority criterion with the fuzzy quantifier “majority”, with the pair (0.3, 0.8) and the corresponding neat
OWA operator with the weighting vector $W_{DM} = \{ 0, 0.4, 0.5, 0.1 \}$, the temporary collective fuzzy preference relation $f^c_{sol}$ is

$$
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5 \\
0.7500 & 0.8331 & 0.7577 & 0.6750 & - \\
0.3211 & 0.2750 & 0.1669 & - & 0.1669 \\
0.2423 & - & 0.3711 & 0.5831 & 0.6789 \\
0.3250 & 0.4169 & 0.6289 & - & - \\
\end{array}
$$

As discussed in Step 4, we obtain the order (Morder) based on the temporary collective solution for finding the proximity derived from QGDD and QGNDD. The order can be determined independently either by QGDD or QGNDD. However, both can also be used with a max operation to determine the order $X_{sol} = \max (QGDD, QGNDD)$.

$$
\begin{align*}
QGDD_{f^c_{sol}} &= \begin{pmatrix} 0.7713 & 0.3013 & 0.4480 & 0.5127 & 0.5466 \end{pmatrix} \\
QGNDD_{f^c_{sol}} &= \begin{pmatrix} 1.0 & 0.7285 & 0.7335 & 0.7938 & 0.8268 \end{pmatrix}
\end{align*}
$$

$$
X_{sol} = \max \{ QGDD_{f^c_{sol}}, QGNDD_{f^c_{sol}} \} \quad (20)
$$

$$
X_{sol} = \begin{pmatrix} 1.0 & 0.7285 & 0.7335 & 0.7938 & 0.8268 \end{pmatrix} \quad (21)
$$

However, for simplicity, we use QGDD to continue with our algorithm, hence $X_{sol}$ will not be used. Similarly, for different factors, weights for the alternatives are determined using the fuzzy quantifier "as many as possible" with the pair (0, 0.5). The corresponding neat OWA operator for the alternative weighting vector is: $W_{Alt} = \{ 0.4, 0.4, 0.2, 0, 0 \}$.

$$
\begin{align*}
QGDD_{DM} &= \begin{pmatrix} DM1 & DM2 & DM3 & DM4 \\
I1 & 0.7447 & 0.7779 & 0.7647 & 0.7779 \\
I2 & 0.3171 & 0.3055 & 0.2971 & 0.3055 \\
I3 & 0.4700 & 0.4263 & 0.4697 & 0.4263 \\
I4 & 0.5800 & 0.5054 & 0.5200 & 0.5054 \\
I5 & 0.6200 & 0.4732 & 0.6200 & 0.4732 \\
\end{pmatrix}
\end{align*}
$$

Based on $QGDD_{DM}$ and $QGDD(f^c_{sol})$, we obtain $M_{order}$, which is used to find the difference of opinions of DM, with $f^c_{sol}$ based on Euclidean distance $(V^i_V^j)$ as described in Step 5.
Now, we estimate the consensus measure ($\zeta$) as described in Step 6 with varying $\lambda$ values ranging from 0.5 to 1.

$$\zeta(a_j) = \begin{pmatrix} \lambda(0.5) & \lambda(0.7) & \lambda(0.9) & \lambda(1) \\ \zeta(I1) & 1 & 1 & 1 \\ \zeta(I2) & 1 & 1 & 1 \\ \zeta(I3) & 1 & 1 & 1 \\ \zeta(I4) & 0.75 & 0.8105 & 0.8566 & 0.875 \\ \zeta(I5) & 0.75 & 0.8105 & 0.8564 & 0.875 \end{pmatrix}$$

Now, we estimate the consensus measure ($\zeta_X$) with neat $\text{OWA}_{\text{LIKE}}$ operator with $\beta = 0.98$ as described in Step 7 to find the $f_{sol}^c$.

$$\zeta_X = \begin{pmatrix} \lambda(0.5) & \lambda(0.7) & \lambda(0.9) & \lambda(1) \\ 0.9163 & 0.9334 & 0.9477 & 0.9539 \end{pmatrix}$$

The final solution $f$ is obtained by combining $I$ (diagonal elements) and $f_{sol}$ (off-diagonal elements). It represents the adoption factors in the form of a complete graph (Figure 3), where vertices and edges are represented by the factors and weights of the collective solution ($f$), respectively. The permanent value of the graph is calculated as follows:
perm(M_{5\times 5}) = \prod_{i} E_i + \sum_{i} \sum_{j} \sum_{k} \sum_{l} \sum_{m} (a_{ij}a_{ji}) E_k E_l E_m \\
+ \sum_{i} \sum_{j} \sum_{k} \sum_{l} \sum_{m} (a_{ij}a_{jk}a_{kl} + a_{ik}a_{kj}a_{ji}) E_l E_m \\
+ \sum_{i} \sum_{j} \sum_{k} \sum_{l} \sum_{m} (a_{ij}a_{jk}) (a_{kl}a_{lk}) E_m \\
+ \left( \sum_{i} \sum_{j} \sum_{k} \sum_{l} \sum_{m} (a_{ij}a_{ji})(a_{kl}a_{lm}a_{mk} + a_{km}a_{ml}a_{lk}) \\
+ \sum_{i} \sum_{j} \sum_{k} \sum_{l} \sum_{m} (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{ml}a_{lk}a_{kj}a_{ji}) \right) \\

where, i, j, k, l and m represents five different factors.

Hence, the final matrix obtained is : 
\[ f^c = I_e + f_{sol}^c \]

\[ f^c = \begin{pmatrix} 0.7073 & - & - & - & - \\ - & 0.3428 & - & - & - \\ - & - & 0.0900 & - & - \\ - & - & - & 0.6708 & - \\ - & - & - & - & 0.3381 \end{pmatrix} + \begin{pmatrix} - & 0.7500 & 0.8331 & 0.7577 & 0.6750 \\ 0.2500 & - & 0.3211 & 0.2750 & 0.1669 \\ 0.1669 & 0.6789 & - & 0.3711 & 0.5831 \\ 0.2423 & 0.7250 & 0.6289 & - & 0.3711 \\ 0.3250 & 0.8331 & 0.4169 & 0.6289 & - \end{pmatrix} \]
The mobile service adoption divide index for the MNO A (Case A) came to be: MSDI (Case A) = 2.237248

It indicates the cumulative impact of all the factors leading to poor usage of mobile services of a particular MNO. This index can be compared with other operators offering similar services, possibly facing a similar set of factors and are offering their services in the same zone. In general, two scenarios for an MNO are never identical from the service provider’s perspective; a particular barrier that acts as a barrier for one particular scenario may not have any impact on the other. Similarly, another operator can also be considered for comparison purpose.

Hence, similar steps were followed to find the index for another operator (Case B) offering mobile services in the same area. The opinion of experts are given below in Table 4:

<table>
<thead>
<tr>
<th>DM1</th>
<th>DM2</th>
<th>DM3</th>
<th>DM4</th>
<th>µi</th>
<th>Entropy(xi)</th>
<th>Cons(xi)</th>
<th>β = 0.0 “At least one of them”</th>
<th>β = 1.0 “Half of them”</th>
<th>β = 100.0 “All of them”</th>
</tr>
</thead>
<tbody>
<tr>
<td>I1</td>
<td>0.9950</td>
<td>0.8650</td>
<td>0.9550</td>
<td>0.7450</td>
<td>0.8900</td>
<td>1.5000</td>
<td>0.8564</td>
<td>0.8900</td>
<td>0.9004</td>
</tr>
<tr>
<td>I2</td>
<td>0.4100</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.5000</td>
<td>0.4775</td>
<td>0.8113</td>
<td>0.9451</td>
<td>0.4775</td>
<td>0.4807</td>
</tr>
<tr>
<td>I3</td>
<td>0.6650</td>
<td>0.5000</td>
<td>0.7450</td>
<td>0.6650</td>
<td>0.6438</td>
<td>1.5000</td>
<td>0.8784</td>
<td>0.6438</td>
<td>0.6561</td>
</tr>
<tr>
<td>I4</td>
<td>0.7450</td>
<td>0.8650</td>
<td>0.7450</td>
<td>0.7450</td>
<td>0.7750</td>
<td>0.8113</td>
<td>0.9262</td>
<td>0.7750</td>
<td>0.7785</td>
</tr>
<tr>
<td>I5</td>
<td>0.6650</td>
<td>0.4100</td>
<td>0.4100</td>
<td>0.5000</td>
<td>0.4963</td>
<td>1.5000</td>
<td>0.8527</td>
<td>0.4963</td>
<td>0.5181</td>
</tr>
</tbody>
</table>

\[
DM1 = \begin{pmatrix}
0.7073 & 0.7500 & 0.8331 & 0.7577 & 0.6750 \\
0.2500 & 0.3428 & 0.3211 & 0.2750 & 0.1669 \\
0.1669 & 0.6789 & 0.0900 & 0.3711 & 0.5831 \\
0.2423 & 0.7250 & 0.6289 & 0.6708 & 0.3711 \\
0.3250 & 0.8331 & 0.4169 & 0.6289 & 0.3381 \\
\end{pmatrix}
\]

\[
DM2 = \begin{pmatrix}
- & 7 & 7 & 5 & 7 \\
1/7 & - & 7 & 1 & 3 \\
1/7 & 1/7 & - & 1/2 & 1/2 \\
1/5 & 1 & 2 & - & 1/2 \\
1/7 & 1/3 & 2 & 2 & - \\
\end{pmatrix}
\]
In this paper, a framework to estimate mobile service adoption divide index is proposed. It uses the perception of telecom experts to derive an index. It helps to quantify the factors systematically with the help of consensus building algorithm. The proposed framework tries to minimize the bias involved when opinions are sought from the experts. The neat OWA helps to aggregate the opinions in a more comprehensive manner. The availability of different input
mechanism puts the experts in a more comfortable position to record their opinions; the transformation functions convert them into a fuzzy preference format, which becomes easier for aggregation using the neat OWA operators.

The use of fuzzy scales offers more flexibility in deciding the importance as it becomes quite difficult to quantify the effects accurately in real situations. The final matrix is a fair representation of the problem under study and offers ease of processing with the help of computer algorithms. Application of digraph and matrix algebra makes it convenient for visual representation and processing of the matrix with the computers. A detailed analysis can be done using MSDI to overcome the factors related to the adoption of mobile services. Resemblance ratio helps to compare two or more different scenarios. The proposed methodology is illustrated with the help of a case study to compare two different MNOs so that a fair judgment can be derived for their capability to overcome the factors.

REFERENCES


Deo, N., 1974. Graph Theory with Applications to Engineering with Computer Science. PHI
Bhadani, Shankar & Rao

Index Development for measuring performance of MNOs

Learning Pvt. Ltd., New Delhi, India.

http://www.dot.gov.in/sites/default/files/3G_Auction_Final_Results_0.pdf


